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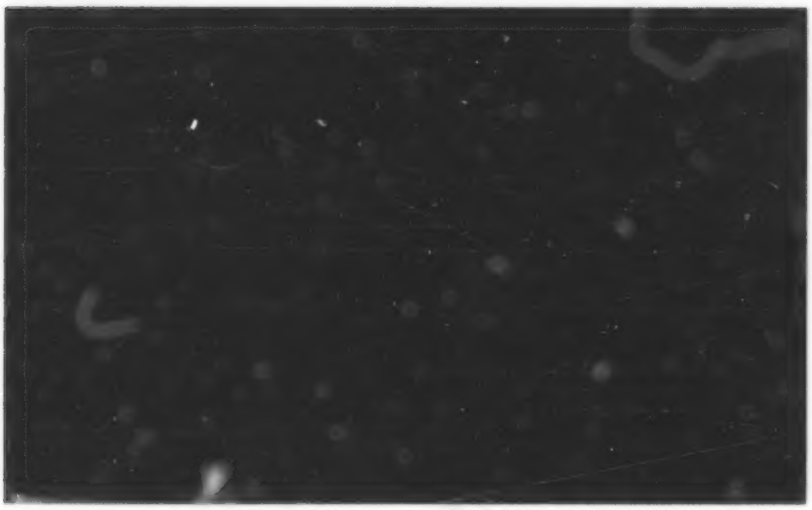

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DOES CONSISTENT AGGREGATION REALLY MATTER?*

by

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ABSTRACT

The problem of determining the existence of consistent input aggregates has been addressed in numerous papers. However, the equivalent problem for output aggregates has not been the subject of many studies. In this paper, we show that existing aggregation tests, mainly developed for inputs, are not suitable for outputs. We propose a more general test which is adequate for inputs and outputs. The proposed test is applied to a sample of Canadian trucking firms in order to find a consistent structure of aggregation for outputs. Our results show that the traditional output aggregate measure used in cost structure analyses of the trucking industry is not consistent. Moreover, using inconsistent aggregation in the cost function specification may alter results on returns to scale and homotheticity of technology.

Key words : aggregation, output, cost function, trucking.

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RÉSUMÉ

Plusieurs études portant sur la détermination de l'existence d'agrégats cohérents des facteurs de production ont été publiées à ce jour. Cependant, bien peu d'études se sont intéressées à la détermination de l'existence d'agrégats cohérents des mesures de production. Dans cette étude, nous montrons que les principaux tests d'agrégation développés à ce jour sont inappropriés pour une application aux mesures de production. Nous proposons un test plus général, à la fois approprié aux facteurs de production et aux mesures de production. Le nouveau test est l'objet d'une application à l'industrie canadienne du transport par camion. Nos résultats montrent que la mesure traditionnelle de production employée lors d'analyses de la structure des coûts des entreprises de transport par camion n'est pas cohérente. Également, nous montrons que l'emploi d'une mesure de production non cohérente peut affecter la mesure des rendements d'échelle de même que les conclusions sur l'homothéticité de la technologie.

Mots-clés : agrégation, production, fonction de coût, camionnage.

INTRODUCTION

The multiproduct nature of technologies employed by most firms is appropriately considered in the traditional framework of the theory of the firm presented in several textbooks.[1] Nevertheless, even if the problem is well understood at the theoretical level, empirical analyses of multiproduct firms have usually neglected the multiproduct aspect of technology. In the best work to date, multiple outputs are aggregated into a unique measure of total output (usually the sum of all outputs) and output quality variables are introduced in order to take into account the multiproduct nature of the technology.[2] In most cases however, the multiple outputs are assumed sufficiently homogeneous to be aggregated and, therefore, considered as a unique output.

To illustrate the multiproduct nature of technologies, let us suppose that two trucking firms, one specialized in less-than-truckload (LTL) shipments and the other in truckload (TL) shipments,[3] both produce the same amount of ton-kilometers. These firms presumably use inputs in different quantities and proportions in order to achieve the same measured level of aggregate production. Consequently, if the total number of ton-kilometers alone is considered as the measure of output, estimates of production or cost structure of the two firms could be misleading.

To avoid such problems, aggregate measures of outputs used in empirical analyses must be consistent. In other words, transformation or cost functions, from which production or cost structure can be infered, must be separable in the output arguments which have been aggregated. Separability of transformation or cost functions is a well known problem. It has been addressed in numerous

papers, both theoretical and empirical, mainly to determine the existence of consistent input aggregates. The determination of consistent output aggregates has long been thought as an equivalent problem and, therefore, did not receive much attention. This paper is an attempt to correct this situation.

However, before determining the existence of consistent output aggregates, it is important to question the empirical relevance of using consistent aggregates rather than aggregates for which consistency is uncertain. This task can be undertaken by comparing estimates of production or cost structure using disaggregated outputs to estimates obtained with aggregated measures of outputs (consistent or not). Thus, after having presented the concept of aggregation or separability in the particular context of production or cost structure analyses in Section 1, Section 2 gives an answer to the title's question by presenting a discussion on the empirical consequences of potentially inconsistent aggregation using trucking industry data. In Section 3, we show that existent input aggregation or separability tests are not suitable for outputs. We then propose a test which is more general than the existent ones in the sense that it is adequate for inputs and outputs. This test is then applied to a sample of Canadian trucking firms in order to find consistent aggregates for the multiple outputs of these firms (Section 4). Concluding comments are presented in Section 5.

1. SEPARABILITY AND CONSISTENT AGGREGATION IN PRODUCTION AND COST STRUCTURE ANALYSES

The use of output aggregates presumably supposes the separability of the transformation function into these aggregates. Let us consider the following

transformation function:

$$f(y,x) \leq 0 \,, \tag{1.1}$$

where y is a vector of output quantities $(y_1,\ldots,y_k,\ldots,y_K)$;

x is a vector of input quantities $(x_1,\ldots,x_m,\ldots,x_M)$;

f is a function satisfying the usual regularity conditions.[4]

In order to define output separability consider first $I = \{1,\ldots,K\}$, the set of indices of the elements of y. Any partition of the elements of y can be described by a corresponding partition of I into a class of mutually exclusive and exhaustive subsets $\{I_1,\ldots,I_n,\ldots,I_N\}$, with $I = I_1 \cup \ldots \cup I_N$ and $I_n \cap I_t = \emptyset$ for $n \neq t$. Consequently, the output bundle $(y_1,\ldots,y_K)$ may be partitioned into $(y^{(1)},\ldots,y^{(N)})$, where $y^{(n)}$ is composed of y_k, $k \in I_n$. Hence, f(y,x) is said to be weakly separable with regard to partition $\{I_1,\ldots,I_N\}$ of y if and only if (Leontief, 1947a, 1947b; Sono, 1961; Green, 1964; Goldman and Uzawa, 1964):

$$\frac{\partial(f_k(y,x) \,/\, f_r(y,x))}{\partial y_i} = 0, \ \forall\, k,r \in I_n \text{ and } i \notin I_n,\ n=1,\ldots,N$$

$$\text{and} \quad \frac{\partial(f_k(y,x) \,/\, f_r(y,x))}{\partial x_m} = 0, \ \forall\, k,r \in I_n,\ n=1,..,N \ \text{and}\ \forall\, m, \tag{1.2}$$

$$\text{where} \quad f_k(y,x) = \frac{\partial f(y,x)}{\partial y_k} \quad \text{and} \quad f_r(y,x) = \frac{\partial f(y,x)}{\partial y_r} \,.$$

f(y,x) is consequently weakly separable with regard to subvectors $y^{(n)}$

(n=1,...,N) when the marginal rates of transformation between any elements of $y^{(n)}$ are independent of elements outside $y^{(n)}$ and also independent of x. In that case, 1.1 can be written as

$$f(y,x) \equiv F(g_1(y^{(1)}),\ldots,g_n(y^{(n)}),\ldots,g_N(y^{(N)}),x) \leq 0$$
$$\equiv F(Y_1,\ldots,Y_n,\ldots,Y_N,x) \leq 0, \qquad (1.3)$$

where $g_1,\ldots,g_N$ are the aggregators;

$Y_1,\ldots,Y_N$ are the aggregates.

Under the assumption that 1.1 satisfies conditions 1.2, consistent aggregates of outputs can be constructed. These correspond to $g_n(y^{(n)})$, n = 1,...,N.

The technological relationship between outputs and inputs defined by 1.1 can be equivalently represented by a cost function given that firms minimize costs and are unable to influence input prices. Considering this technological relationship, the cost function is given by

$$c(y,w) \equiv \min_x\{\, w'x : f(y,x) \leq 0\}, \qquad (1.4)$$

where w is a vector of input prices $(w_1,\ldots,w_n,\ldots,w_N)$;

c is a nondecreasing, homogeneous of degree one, and

concave function in w, and also strictly increasing in y.

If the transformation function satisfies conditions 1.2 then, by the envelope theorem, the cost function defined by 1.4 also satisfies the

separability conditions[5]

$$\frac{\partial(c_k(y,w) \,/\, c_r(y,w))}{\partial y_i} = 0, \ \forall\, k,r \in I_n \text{ and } i \notin I_n,\ n=1,\ldots,N$$

$$\text{and} \quad \frac{\partial(c_k(y,w) \,/\, c_r(y,w))}{\partial w_m} = 0, \ \forall\, k,r \in I_n,\ n=1,..,N \text{ and } \forall\, m, \qquad (1.5)$$

$$\text{where} \quad c_k(y,w) = \frac{\partial c(y,w)}{\partial y_k} \quad \text{and} \quad c_r(y,w) = \frac{\partial c(y,w)}{\partial y_r}\,.$$

Therefore, $c(y,w)$ can be written as

$$\begin{aligned} c(y,w) &\equiv C(g_1(y^{(1)}),\ldots,g_n(y^{(n)}),\ldots,g_N(y^{(N)}),w) \\ &\equiv C(Y_1,\ldots,Y_n,\ldots,Y_N,w). \end{aligned} \qquad (1.6)$$

The above discussion introduced separability and consistent aggregation in terms of the transformation function or, its dual representation, the cost function. Let us now assess the empirical importance of considering output aggregates for which consistency has been determined.

2. THE RELEVANCE OF CONSIDERING CONSISTENT AGGREGATES

Suppose that instead of $c(y,w)$ we are considering $C(Y_1,\ldots,Y_n,\ldots,Y_N,w)$, an aggregated version of the cost function, under the assumption that $c(y,w) \equiv C(Y_1,\ldots,Y_n,\ldots,Y_N,w)$. What are the empirical consequences of output aggregation

in the case where the above assumption is false? In other words, does inconsistent aggregates yield false results on the production or cost structure?

To answer this question, we consider two different cost functions which have been estimated with a sample of Canadian trucking firms.[6] The specification employed for the first cost function (disaggregated) is[7]

$$TC = c(y,w,q,T), \tag{2.1}$$

where TC are total costs;

q is a vector of technological characteristics ($q_1,\ldots,q_4,\ldots,q_{11}$);

T is a time trend.

Our choice of output measures has been dictated by the data availability and the nature of the technology employed by trucking firms. Thus, a weight-distance classification is used for less-than-truckload ($\leq$ 5000 kg.) and truckload (> 5000 kg.) shipments and for short, medium, and long haul shipments.

Technological characteristic variables are used to represent the network over which the firm operates (see Gagné, 1990 for a discussion on the interpretation of cost elasticities with respect to technological characteristic variables). Hence, q_1 (average length of haul), q_2 (average shipment size), and q_3 (average load per truck) are all proxies for network size and density. The insurance variable (q_4) is a proxy which take into account the specific mix of commodities carried by each firm. A higher value for this variable represents a larger proportion of more valuable or more fragile goods carried by the firm. The

time trend has been included to measure potential displacements of the cost function through time due to technical change or to the yearly changes in the sample's composition of firms.

The specification of the second cost function (fully aggregated) is

$$TC = C(Y,w,q,T), \tag{2.2}$$

where $Y = \sum_{k=1}^{K} y_k$.

Note that specification 2.2 has been used in most of the empirical studies of the technology and cost structure of the trucking industry.[8] There is no evidence at this point supporting either the consistency or the inconsistency of Y. Both cost functions have been approximated by a second-order translog form which represents a second-order Taylor series approximation to an unknown function.[9] They have been estimated with their corresponding input share equations, using the iterative version of the Zellner (1962) procedure.[10] Moreover, since input shares sum to 1, an input share equation was deleted in the estimation procedure in order to avoid the singularity of the estimated disturbance covariance matrix.

The left-hand side of table 1 reports the first-order parameter estimates of the disaggregated specification and the right-hand side those of the fully aggregated one.[11]

INSERT TABLE 1 HERE

Since our main concern is the aggregation of output measures, we will focus the discussion on the effects of outputs on total costs.[12] That is, we will verify what type of returns to scale characterized the technology of trucking firms. To measure returns to scale in a multiproduct world, we will use the following measure proposed by Baumol, Panzar and Willig (1982):

$$S(y,w,q,T) = 1 \Big/ \sum_{k=1}^{K} \left(\partial \ln c(y,w,q,T) / \partial \ln y_k \right). \qquad (2.3)$$

For the single output case 2.3 becomes

$$S(Y,w,q,T) = 1 \Big/ \left(\partial \ln C(Y,w,q,T) / \partial \ln Y \right). \qquad (2.4)$$

Consequently, when $S < 1$ returns to scale are said to be decreasing, whereas $S > 1$ indicates increasing returns to scale. $S = 1$ stands for constant returns to scale.

Let us now consider the results obtained with the disaggregated specification. At the point of approximation ($y = y^0$, $w = w^0$, $q = q^0$, $T = T^0$), constant returns to scale cannot be rejected at any reasonable confidence level (see table 2 for a summary of the tests performed). However, this is a local measure of returns to scale and it does not allow us to infer returns to scale elsewhere unless the technology is homothetic. In that particular case, returns to scale, as defined by 2.3, are independent of w and q, and therefore can be characterized globally by the ray average cost function introduced by Baumol, Panzar and Willig (1982, chap. 3). Since our disaggregated cost function has been approximated by a second-order translog form, testing for homotheticity is a

rather simple task. The technology is said to be homothetic if

$$\frac{\partial^2 \ln c(y,w,q,T)}{\partial \ln y_k \, \partial \ln w_b} = 0, \quad \forall \text{ k and m,}$$

$$\frac{\partial^2 \ln c(y,w,q,T)}{\partial \ln y_k \, \partial \ln q_u} = 0, \quad \forall \text{ k and u.} \qquad (2.5)$$

We have imposed restrictions 2.5 to the approximation of the disaggregated cost function and re-estimated it. Results presented in table 2 show that homotheticity can be rejected even at a 1% confidence level. Consequently, returns to scale cannot be characterized globally here.

Results obtained with the fully aggregated specification are clearly differents. Constant local returns to scale (at $Y = Y^0$, $w = w^0$, $q = q^0$ and $T = T^0$) are rejected at a 10% confidence level but are not at a 1% level. Since a 10% confidence level is often considered as satisfactory, one must conclude that the technology is not characterized by constant returns to scale, which contradicts the preceding results. In addition, a test for homotheticity has been performed and did not rejected it even at a 10% confidence level. This again contradicts results obtained with the disaggregated specification.

INSERT TABLE 2 HERE

The passage from a disaggregated specification to a fully aggregated one seems to have an important impact on the technology or cost structure that can be infered from those specifications. However, this impact can be attributable not only to aggregation but also to the aggregator used (here the sum of all outputs). In the following sections, we will see that aggregation is, at least partly, responsible for the bias that have been introduced.

3. TESTING THE EXISTENCE OF CONSISTENT OUTPUT AGGREGATES

The empirical determination of the existence of consistent output aggregates is possible if one could verify that the transformation function (or the cost function) satisfies conditions 1.2 (1.5). Denny and Fuss (1977) have proposed a test which aims to determine the existence of consistent input aggregates for a cost function approximated by a second-order translog form. To summarize their test, consider $c^*(y,w,a)$ a twice continuously differentiable approximation of $c(y,w)$, where a stands for the vector of parameters to be estimated.[13] Once $c^*(y,w,a)$ has been estimated by an adequate econometric method, conditions 1.5 can be verified by placing the appropriate restrictions on the elements of a.

This procedure implicitly assumes that the approximation's derivatives (evaluated at the point of approximation) are good predictors of the derivatives of the true underlying cost function $c(y,w)$. However, following White (1980) and Coleman (1985), it seems unappropriate to consider that it is the case.

Partly in response to the critic addressed by White (1980) and Coleman

(1985), Blackorby, Schworm and Fisher (1987) have recently proposed a new procedure for testing the existence of consistent input aggregates in a cost function. Using their procedure in the present context would first imply to estimate the parameters of an approximation of c(y,w) and then find and test the appropriate set of restrictions on these parameters which would make this approximation equivalent to an approximation of $C(Y_1,\ldots,Y_N,w)$.

In this paper we present a different approach. Consider $h^*_{kr}(y,w,e)$, an at least once continuously differentiable flexible[14] approximation of $c_k(y,w)/c_r(y,w)$ which insures its theoretical properties[15] (e is a disturbance term which stands for approximation error, E(e)=0 and $Var(e)=\sigma_e^2$). Given conditions 1.5, c(y,w) can be written $C(Y_1,\ldots,Y_n,\ldots,Y_N,w)$ when

$$\frac{\partial h^*_{kr}(y,w,e)}{\partial Y_i} = 0, \ \forall\, k,r \in I_n \text{ and } i \notin I_n,\ n=1,\ldots,N$$

and

$$\frac{\partial h^*_{kr}(y,w,e)}{\partial w_m} = 0, \ \forall\, k,r \in I_n \text{ and } \forall\, m., \ n=1,\ldots,N \qquad (3.1)$$

Testing separability, and consequently the existence of consistent output aggregates, is therefore testing whether the ratio of two marginal cost functions is affected by variables which are not part of the considered aggregate. One must therefore obtain estimates of $h^*_{kr}(y,w,e)$ and then verify conditions 3.1.

This test is different from the existent ones because it is peformed on approximations of marginal cost function ratios rather than on the ratio of marginal cost functions approximated by the derivatives of the approximation of c(y,w). In addition, this test does not requires assumption about the specific

form of the aggregators. However, the level of $c_k(y,w)/c_r(y,w)$ must be observed, otherwise the estimation of $h^*_{kr}(y,w,e)$ is not possible. On the other hand, applying Shephard's lemma, this test is already adequate for the determination of consistent input aggregates.

There exists several situations where marginal costs or marginal cost ratios can be observed when market prices are observed. For instance, perfectly competitive markets imply equality between price and marginal cost on those markets, consequently

$$\frac{p_k}{p_r} = \frac{c_k(y,w)}{c_r(y,w)} = h^*_{kr}(y,w,e), \qquad (3.2)$$

where p_k and p_r are respectively prices on markets k and r, two perfectly competitive markets. A test on the separability of c(y,w) could therefore be performed.

Even if the markets are imperfectly competitive, equality 3.2 can still be valid when industry conditions are such that the permanent threat of entry by potential competitors prevent existing firms to significantly deviate from a price equals marginal cost equilibrium. Again here, given that market prices are observed, marginal costs are also observed.

When markets are imperfectly competitive and when there is no permanent threat of entry by potential competitors, equality 3.2 is no longer valid. However, over the last decade, a new literature on the estimation of price-cost margins has emerged. This literature has been recently surveyed by Bresnahan

(1989). Consequently, given that price-cost margins can be adequately estimated, approximations of $c_k(y,w)/c_t(y,w)$ can also be estimated, regardless of market conditions.

4. EMPIRICAL ANALYSIS: TESTING THE EXISTENCE OF CONSISTENT OUTPUT AGGREGATES IN THE CANADIAN TRUCKING INDUSTRY

Empirical studies of the cost structure of both canadian and U.S. trucking industries have been numerous.[16] Most of them were using a unique aggregate measure of output (total ton-kilometers carried by the firm) and were taking into account the multiproduct nature of the technology employed by trucking firms by introducing technological characteristic or output quality variables into the cost function (specification 2.2). As we have seen, using such an aggregate measure can seriously alter statistical results. There is two reasons that can explain this outcome: i) there exists no consistent aggregate measure of all outputs or ii) even if there exists a unique measure of all outputs that is consistent, the aggregator used could be inadequate. By performing the test proposed above, one can determine if a consistent aggregate measure exists. If it does not, the test permits to find other structures of aggregation that could be consistent. If it does, one must then find an aggregator which is adequate.

In order to find consistent output aggregates for trucking firms using the above test we can hardly assume perfect competition given that firms in our sample are regulated (except, of course, if the regulator succeed in implementing conditions equivalent to perfect competition, which is a rather strong assumption). It is also beyond the scope of this paper to estimate price-cost margins. Nevertheless, given that firms in our sample are amongst the largest

firms in the industry,[17] most of them are therefore in direct competition on intra- and inter-provincial markets as well as on international markets. They thus face the same regulatory environment and also the same type of demand from shippers. Since there is no reason a priori to believe that regulation can affect them differently, they should enjoy the same market power as measured by the Lerner index.

The Lerner index on market k is given by

$$l_k = \frac{p_k - mc_k}{p_k}, \tag{4.1}$$

where $mc_k = c_k(y,w)$, the marginal cost function with respect to y_k.

Under the assumption that market power as measured by l_k is not firm-specific, we have that

$$\frac{p_k}{p_r}(L_{kr}) = c_k(y,w)/c_r(y,w), \tag{4.2}$$

where $L_{kr} = (1 - l_k)/(1 - l_r)$, a constant for all firms in the sample.

Since the left-hand side of 4.2 is multiplied by a constant, approximations of the marginal cost ratios can be estimated using price ratios and, therefore, conditions 3.1 can be adequately tested.

In order to test for several partitions of the output vector y, second-

order translog approximations of the following equations were estimated by ordinary least squares:[18,19]

$$\frac{p_k}{p_r} = h^*_{kr}(y,w,q,T,e), \text{ for } k,r=1,\ldots,6 \text{ and } k<r. \qquad (4.3)$$

For each partition considered, the following conditions were tested (using a F test) at the point of approximation:

$$\frac{\partial h^*_{kr}(y,w,q,T,e)}{\partial y_i} = 0, \ \forall\, k,r \in I_n \text{ and } i \notin I_n, \ n=1,\ldots,N, \text{ and } k<r,$$

$$\frac{\partial h^*_{kr}(y,w,q,T,e)}{\partial w_m} = 0, \ \forall\, k,r \in I_n, \ n=1,\ldots,N, \ \forall\, m, \text{ and } k<r,$$

$$\frac{\partial h^*_{kr}(y,w,q,T,e)}{\partial q_u} = 0, \ \forall\, k,r \in I_n, \ n=1,\ldots,N, \ \forall\, u, \text{ and } k<r,$$

and

$$\frac{\partial h^*_{kr}(y,w,q,T,e)}{\partial T} = 0, \ \forall\, k,r \in I_n, \ n=1,\ldots,N, \text{ and } k<r. \qquad (4.4)$$

Table 3 summarizes the results obtained. Six partitions of y were considered. The first partition corresponds to a fully aggregated structure while the others are partitions based on weight or distance of shipments. Results reported in table 3 show that a consistent aggregate measure of all the outputs does not exist. Therefore, the impact of aggregation on statistical results discussed in Section 2 seems to be, at least partially, attributable to inconsistent aggregation. In other words, the cost function is not separable with respect to a partition formed by all the output measures, thereby a consistent aggregate of these measures cannot be constructed. We must therefore find a

structure of aggregation for which consistency can be established.

Results presented in table 3 indicate that two of the six considered partitions cannot be rejected as separable partitions of y. Both preserve a distinction between LTL and TL shipments but inside each of these categories aggregation seems possible. Thus, our results show that LTL shipments of medium and long haul can be aggregated and that TL shipments of short and medium haul can also be aggregated. In addition, another unrejected partition admits aggregation between short and medium haul shipments in the LTL segment and the same pattern of aggregation in the TL segment. Overall, the unrejected partitions are intuitively interesting. They both make a distinction between LTL and TL shipments (which constitute two dramatically different types of shipments) and they both admit aggregation on the ground of distance.

INSERT TABLE 3 HERE

To compare the results obtained precedently with the disaggregated specification to what could be obtained from a specification which contains consistent output aggregates, we have estimated[20] the second-order translog approximation of

$$TC = \hat{C}(Y_1, Y_2, Y_3, Y_4, w, q, T), \qquad (4.5)$$

where $Y_1 = y_1 + y_2$;

$Y_2 = y_3$;

$Y_3 = y_4 + y_5$;

$Y_4 = y_6$.

Note that the stucture of aggregation used in 4.5 is consistent, although there is no evidence that the aggregators used are adequate. The first-order parameter estimates of this specification are presented in table 4.[21] Constant returns to scale at the point of approximation cannot be rejected for this specification as reported in table 5. In addition, homotheticity of the technology is rejected implying that returns to scale cannot be characterized globally. Equivalent results were obtained with the other non-rejected partition.

INSERT TABLES 4 AND 5 HERE

These results are in accordance to those obtained with the disaggregated specification, indicating that the partitions found by our test seem to give consistent aggregates. Finally, it seems that the aggregators used here (the sum of outputs) did not introduce any bias, at least for results on returns to scale.

5. CONCLUDING COMMENTS

In this study, we have proposed a test which can determine the existence of consistent output aggregates. This test has been applied to a sample of

canadian trucking firms and it has been shown that the traditional output measure used in cost structure analyses of this industry is not consistent.

This study also showed that non consistent output aggregates alter seriously the results of cost structure estimates. For instance, in the trucking industry, when the traditional output measure is used in a cost function specification, results on returns to scale and homotheticity are affected. Alternative consistent output aggregates have been found. Their use in cost function specifications did not seem to create any bias in the estimates.

The proposed test has been applied under the assumption that all firms enjoy the same market power. It would be interesting to extend its application to other types of equilibria by first estimating the gaps between prices and marginal costs for each firm in the sample and then use this additional information to test for the existence of consistent output aggregates regardless of market conditions.

Notes

1- See, for instance, Chambers (1988), chap. 7 or Nadiri (1982).

2- This procedure has been employed notably for cost structure analyses of the trucking industry (Spady and Friedlaender, 1978, 1981) and also for similar analyses of the airline industry. In addition, an attempt has been made by Friedlaender and Wang Chiang (1984) to determine the existence of consistent output aggregates for trucking firms. The present paper will discuss the inadequacy of the procedure they employed.

3- See Diewert (1982), McFadden (1978) or Lau (1978). In addition, f must be strictly increasing in y.

3- LTL shipments usually require consolidation at a terminal while TL shipments do not.

5- Since $c(y,w) = \min_x \{ w'x : f(y,x) \leq 0 \}$ then, by the envelope theorem,

$$\frac{\partial c(y,w)}{\partial y_k} = - \lambda^* \frac{\partial f(y,x^*)}{\partial y_k}, \quad k=1,\ldots,K,$$

where stars symbolise optimisers and λ the Lagrange multiplier.

From this, conditions 1.5 follow.

6- The estimations were undertaken using a sample of 494 firms. Supplementary details about this sample could be found in the appendix.

7- See the appendix for a complete description of the variables.

8- See, for instance, Spady and Friedlaender (1978,1981), Friedlaender and Wang Chiang (1983), Kim (1984), Friedlaender and Schur Bruce (1985), Daughety, Nelson and Vigdor (1985), Daughety and Nelson (1988), and Gagné and Dionne (1988).

9- The translog form (second-order) of a given function $a=h(b)$, $b=(b_1,\ldots,b_i,\ldots,b_I)$ is

$$\ln a = \alpha_0 + \sum_i \alpha_i \ln(b_i/b_i^0) + \tfrac{1}{2} \sum_i \sum_j \alpha_{ij} \ln(b_i/b_i^0) \ln(b_j/b_j^0),$$

where b_i^0 and b_j^0, $i,j=1,\ldots,I$, represent the point of approximation.

10- Homogeneity and symmetry restrictions have been imposed prior estimation.

11- Complete results are available from the author upon request.

12- Of course, inconsistent aggregation could affect other aspects of the technology.

13- Since the second-order translog approximation of $c(y,w)$ represents a second-order Taylor series approximation of the logarithm of this function, the vector of parameters a could be interpreted as the value, first and second derivatives of the logarithm of $c(y,w)$ evaluated at the point of approximation.

14- We use here the Diewert's definition of flexibility (Diewert, 1971,1973).

15- For instance, since $c(y,w)$ is homogeneous of degree one in w then $c_k(y,w)$ and $c_r(y,w)$ are also homogeneous of degree one in w and, therefore, $c_k(y,w)/c_r(y,w)$ is homogeneous of degree zero in w. Also, since $c(y,w)$ is strictly increasing in y then $h^*_{kr}(y,w,e)$, $k,r=1,\ldots,K$, are strictly positive functions. However, the concavity of $c(y,w)$ in w does not imply any particular form for $h^*_{kr}(y,w,e)$.

16- See note 8.

17- Sample's average total cost is \$23 millions. In addition, most of the firms reporting data to Statistics Canada are class I and II firms, which are the biggest firms in terms of revenues.

18- $h^*_{kr}(y,w,q,T,e)$ is an approximation of $c_k(y,w,q,T)/c_r(y,w,q,T)$, where

$$c_k(y,w,q,T) = \frac{\partial c(y,w,q,T)}{\partial y_k} \quad \text{and} \quad c_r(y,w,q,T) = \frac{\partial c(y,w,q,T)}{\partial y_r}.$$

Again, e stands here for approximation error.

19- A second-order approximation of $c_k(y,w,q,T)/c_r(y,w,q,T)$ corresponds to a third-order approximation of $c(y,w,q,T)$. Although we have precedently used a second-order approximation of the cost function, we use here second-order instead of first-order approximations of $c_k(y,w,q,T)/c_r(y,w,q,T)$ in order to obtain the best possible estimates of these ratios.

20- Using the same method as precedently and placing the appropriate restrictions.

21- See note 11.

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APPENDIX

1) Data Sources

Data used in this study come from two surveys realized by Statistics Canada: the Motor Carrier Freight survey (MCF) and the Origin-Destination Trucking survey (TOD). The MCF survey contains information on trucking firms. It includes the financial statement as well as data on employment, fuel consumption, equipment, etc. The TOD survey contains a sample of shipments carried by a subset of firms of the MCF survey. For each shipment in the sample, information on the origin, the destination, the type of commodity, the weight, and the revenue generated by the shipment is available.

We have first merged these two data sources and then kept only those firms which were respecting the following criterias:

- General freight must have accounted for at least 50% of total revenues. This is to preserve a relative technological homogeneity between selected firms.

- Each firm must have purchased each of the four inputs (see below) and must have declared strictly positive total costs, input prices, and technological characteristics.

- Finally, each element of the output vector y must have been strictly positive.

Given these criterias, a sample of 494 observations has been created. Selected firms come from all provinces. The observation period is 1981-1986. Thus, for some selected firms, data are available for more than one year.

2) Variable Description

TC : Total operating costs (including a 10% opportunity cost)
(TC^0 = \$ 23 041 574.60);

w_1 : Price of fuel (including taxes)
(w_1^0 = \$ 0.39/litre);

w_2 : Price of labour (drivers and helpers, including fringe benefits)
(w_2^0 = \$ 29 526.36/man-year);

w_3 : Price of capital
(w_3^0 = \$ 0.98/\$ unit of capital stock);

w_4 : Price of materials (a Divisia index of the prices of all other inputs)
(w_4^0 = \$ 6 749.78);

y_1 : Total ton-kilometers of class 1 shipments ($\leq$ 5000 kg. and $\leq$ 400 km.)
(y_1^0 = 3 797 998 t-k);

y_2 : Total ton-kilometers of class 2 shipments ($\leq$ 5000 kg. and > 400 km. but $\leq$ 800 km.)
(y_2^0 = 6 760 792 t-k);

y_3 : Total ton-kilometers of class 3 shipments ($\leq$ 5000 kg. and > 800 km.)
(y_3^0 = 20 428 953 t-k);

y_4 : Total ton-kilometers of class 4 shipments (> 5000 kg. and $\leq$ 400 km.)
(y_4^0 = 18 597 259 t-k);

y_5 : Total ton-kilometers of class 5 shipments (> 5000 kg. and > 400 km. but $\leq$ 800 km.)
(y_5^0 = 29 523 136 t-k);

y_6 : Total ton-kilometers of class 6 shipments (> 5000 kg. and > 800 km.)
(y_6^0 = 62 692 011 t-k);

p_k : Average firm-specific price per ton-kilometer for class k shipments (k=1,...,6) with

$p_{1_0}^0$ = \$ 0.7390/ton-kilometer;
$p_{2_0}^0$ = \$ 0.4580/ton-kilometer;
$p_{3_0}^0$ = \$ 0.3270/ton-kilometer;
$p_{4_0}^0$ = \$ 0.1410/ton-kilometer;
$p_{5_0}^0$ = \$ 0.0790/ton-kilometer;
p_6^0 = \$ 0.0590/ton-kilometer;

q_1 : Average length of haul
(q_1^0 = 641.46 km.);

q_2 : Average shipment size
(q_2^0 = 5.24 tons);

q_3 : Average load per truck
(q_3^0 = 0.05 ton/vehicle-kilometer);

q_4 : Insurance for losts and damages
(q_4^0 = \$ 0.0014/ton-kilometer);

T : Time trend, T=1 for 1981,..., T=6 for 1986
(T^0 = 3.52);

where TC^0, y^0, p^0, w^0, q^0, and T^0 stand for the sample mean of the variables.

Table 1

Cost Functions First-Order Parameters Estimates
Disaggregated and Aggregated Specifications
(t-ratios in parentheses)

Disaggregated		Aggregated	
Variable	Parameter	Variable	Parameter
Constant	17.1084**	Constant	16.9171**
	(141.74)		(306.82)
y_1	0.0614	$Y=\sum y_k$	0.9546**
	(1.02)		(37.42)
y_2	0.0187	-	-
	(0.03)		
y_3	0.1287*	-	-
	(2.62)		
y_4	0.1285	-	-
	(1.68)		
y_5	0.1985*	-	-
	(3.19)		
y_6	0.04315**	-	-
	(5.93)		
w_1	0.0573**	w_1	0.0610**
	(12.18)		(16.76)
w_2	0.1403**	w_2	0.1369**
	(17.78)		(23.13)
w_3	0.2403**	w_3	0.2534**
	(29.27)		(41.81)
w_4	0.5621**	w_4	0.5487**
	(41.02)		(54.36)
q_1	-0.3022	q_1	-0.2081**
	(-1.49)		(-3.34)
q_2	0.0910	q_2	0.0072
	(0.73)		(0.20)
q_3	-0.1416*	q_3	-0.0946*
	(-1.97)		(-2.27)
q_4	0.3426**	q_4	0.2760**
	(6.26)		(7.63)
T	-0.0294	T	-0.0586
	(-0.33)		(-0.94)
R^2	0.82		0.86
Number of estimated parameters	120		55
Log(L)	2164.53		2122.27

* Statistically significant at a 5% confidence level
** Statistically significant at a 1% confidence level

Table 2

Summary of Statistical Tests
Constant Returns to Scale and Homotheticity

Null hypothesis	Log(L)	Confidence level	Number of restrictions	Critical value(χ^2)	Computed statistic	Rejection of the null hypothesis
i) Disaggregated Specification						
No rest.	2164.53	-	-	-	-	-
C.R.S.	2164.18	1%	1	6.63	0.70	No
		10%	1	2.71	0.70	No
Homo.	2103.06	1%	42	63.70	122.94	Yes
ii) Aggregated Specification						
No rest.	2122.27	-	-	-	-	-
C.R.S.	2120.82	1%	1	6.63	2.90	No
		10%	1	2.71	2.90	Yes
Homo.	2120.27	10%	7	12.02	4.00	No

Note: All tests are likelihood ratio tests. Thus, the non-restricted values of the log-likelihood functions have been compared to the restricted ones.

Table 3

Summary of Tests on Separability

Partition	h_{kr} Function	Computed F	Critical Values at 1%	at 5%	at 10%	Partition Rejected
$(y_1,y_2,y_3,$ $y_4,y_5,y_6)$	k=1,r=2	2.0907				
	" ,r=3	3.0117				
	" ,r=4	4.4273				
	" ,r=5	4.9200				
	" ,r=6	3.4992				
	k=2,r=3	0.9513				
	" ,r=4	5.2050				
	" ,r=5	5.5967	2.5590	1.9632	1.6867	yes
	" ,r=6	3.9880				
	k=3,r=4	3.8664				
	" ,r=5	3.1838				
	" ,r=6	3.7948				
	k=4,r=5	1.6856				
	" ,r=6	2.0194				
	k=5,r=6	1.0900				
(y_1,y_2,y_3) (y_4,y_5,y_6)	k=1,r=2	1.6495				
	" ,r=3	2.5354				
	k=2,r=3	0.8550	2.2954	1.8143	1.5879	yes
	k=4,r=5	1.2999				
	" ,r=6	3.2223				
	k=5,r=6	2.3804				
$y_1,(y_2,y_3)$ $(y_4,y_5),y_6$	k=2,r=3	0.8023	2.2325	1.7781	1.5635	no
	k=4,r=5	1.3040				
$(y_1,y_2),y_3$ $(y_4,y_5),y_6$	k=1,r=2	1.5140	2.2325	1.7781	1.5635	no
	k=4,r=5	1.3040				
$(y_1,y_2,y_4,$ $y_5),y_3,y_6$	k=1,r=2	1.6732				
	" ,r=4	4.2892				
	" ,r=5	4.1810	2.3685	1.8560	1.6158	yes
	k=2,r=4	4.5985				
	" ,r=5	4.6924				
	k=4,r=6	1.4839				
(y_1,y_4)	k=1,r=4	4.8419				
(y_2,y_5)	k=2,r=5	4.4471	2.2325	1.7781	1.5635	yes
(y_3,y_6)	k=3,r=6	2.9773				

Table 4

Cost Functions First-Order Parameters Estimates
Consistently Aggregated Specification
(t-ratios in parentheses)

Variable	Parameter
Constant	17.0711**
	(150.66)
$Y_1=y_1+y_2$	0.0097
	(0.11)
$Y_2=y_3$	0.1383**
	(3.06)
$Y_3=y_4+y_5$	0.3421**
	(4.12)
$Y_4=y_6$	0.4750**
	(7.38)
w_1	0.0530**
	(10.44)
w_2	0.1377**
	(16.43)
w_3	0.2389**
	(27.73)
w_4	0.5705**
	(39.51)
q_1	-0.3109*
	(-2.14)
q_2	-0.0145
	(-0.11)
q_3	-0.1596**
	(-2.58)
q_4	0.3258**
	(6.40)
T	0.0103
	(0.12)
R^2	0.84
Number of estimated parameters	91
Log(L)	2167.17

* Statistically significant at a 5% confidence level
** Statistically significant at a 1% confidence level

Table 5

Summary of Statistical Tests
Constant Returns to Scale and Homotheticity
Consistently Aggregated Specification

Null hypothesis	Log(L)	Confidence level	Number of restrictions	Critical value(χ^2)	Computed statistic	Rejection of the null hypothesis
No rest.	2167.17	-	-	-	-	-
C.R.S.	2166.61	1%	1	6.63	1.12	No
		10%	1	2.71	1.12	No
Homo.	2117.18	1%	28	48.28	99.98	Yes

Note: All tests are likelihood ratio tests. Thus, the non-restricted value of the log-likelihood function has been compared to the restricted ones.